Problem Set

MA18Q3-G

mail@**kenjisato**.jp

Day 10

Cake eating problem

You have $w(0)$ kilogram of cake at time $t = 0$. The amount of cake at t, $w(t)$, follows the differential equation

$$\dot{w}(t) = -c(t),$$

where $c(t)$ [kg/min] is the instantaneous speed of consumption at time t. Find a consumption stream $c(t)$ that maximizes your utility,

$$U = \int_0^\infty e^{-\rho t} \ln c(t) dt,$$

where $\rho > 0$ is a constant discount rate.

1. Set up the Hamiltonian for the problem.
2. Derive the differential equation that c obeys.
3. Use $w(0) = \int_0^\infty c(t)dt$, which states that you are going to eat up the whole cake, to fully determine $c(t)$. [Optimality requires a condition similar to this one.]