Problem Set

MA18Q3-E

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Day 4

[1] Basic properties of growth rates.

Romer 4e, Problem 1.1. The growth rate of a variable equals the time derivative of its log, i.e. $\dot{X}(t)/X(t) = \frac{d}{dt}[\ln X(t)]$, where $\dot{X}(t) = \frac{dX}{dt}(t)$. Use this fact to show:

1. If $Z(t) = X(t)Y(t)$, then $\dot{Z}(t)/Z(t) = [\dot{X}(t)/X(t)] + [\dot{Y}(t)/Y(t)]$.
2. If $Z(t) = X(t)/Y(t)$, then $\dot{Z}(t)/Z(t) = [\dot{X}(t)/X(t)] - [\dot{Y}(t)/Y(t)]$.
3. If $Z(t) = X(t)^\alpha$, then $\dot{Z}(t)/Z(t) = \alpha\dot{X}(t)/X(t)$.

[2] Application of the growth rate formulas

Jones 2017, p. 67. Suppose that x and y grow at constant nominal rates given by 0.04 and 0.02. Calculate the growth rate of z in each of the following cases.

1. $z = xy$
2. $z = x/y$
3. $z = y/x$
4. $z = x^{1/2}y^{1/2}$
5. $z = (x/y)^2$
6. $z = x^{-1/3}y^{2/3}$

[3] Estimation

Let $y = Ak^\alpha$. We observe that $\alpha = 0.3$ and y and k grow at a constant annual rate of 0.05 and 0.1, respectively. Estimate the growth rate of A.

TIPS

Recall that the (average) nominal growth rate can be estimated by

$$\begin{aligned} g_X &:= \log X(t+1) - \log X(t) \\ g_Y &:= \log Y(t+1) - \log Y(t). \end{aligned}$$

The nominal growth rate, g_{XY}, for XY is

$$\begin{aligned} g_{XY} &= \log X(t+1)Y(t+1) - \log X(t)Y(t) \\ &= [\log X(t+1) - \log X(t)] + [\log Y(t+1) - \log Y(t)] \\ &= g_X + g_Y. \end{aligned}$$